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The Mathematics Teacher's Beliefs about Mathematics Knowledge and Its Teaching: A Case Study

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Abstract: The aim of this study is to present the insights of the mathematics teachers beliefs' on mathematical knowledge, and its teaching and learning. The case study method is used in this study. tThe study pursued along with the participation of one secondary mathematics teacher in the fall semester of 2007-2008 instruction year in Trabzon providence. The interviews realized during approximately 45 minutes with the interwiewee for the four times. In the interviews, three data collection forms were used as the data collection tool. These are: a) Semi-structured interview form, b) Hypothetical situations called "episodes" as a means to initiate responses from the informant, c) Documents that are including pictures reflecting different classroom settings and about the nature of mathematical knowledge. By using Magolda's Epistemological Reflection Model, the data obtained were analyzed descriptively in a qualitative manner. Besides holding some strict beliefs on nature of mathematical knowledge, evaluation, and mathematical teaching, participant have some flexible beliefs about mathematics and its teaching,in addition, it was concluded that participant open to some new perceptions. However, we believe that by conducting similar studies with larger samples it might be possible to contribute to teacher training researches in our country.

Keywords: mathematics teachers' beliefs, effective teaching, teacher training, reform effort in education

Introduction

Certain targets are set in all stages of educational systems in order to increase the quality of mathematics education. Some of these targets that students are expected to reach could be listed as to comprehend mathematical concepts, to gain problem solving skills, to feel confidence about mathematics, and to have positive attitude towards mathematics. Certain factors affect whether these targets are reached. Unquestioningly, one of these factors is the beliefs that teachers have about the nature and instruction of mathematics (Baydar & Bulut, 2002). According to the related literature, teacher beliefs give shape to instructional activities in classroom settings and that classroom activities which are observable in the classroom are actually the results of the unobservable process proceeding in teacher's mind (Boonyaprakob, 2002). For the new educational notion, Yılmaz (2004) suggested that the teaching activities should be based on understanding, analyzing, synthesizing, open and scientific thought instead of basic knowledge transfer and memorization. He also argued that to overcome such a fundamental change in the educational system, the most important task and responsibility pertains to the teachers,and 'change' must take place in teachers' mind at first. Taking the issue from this point of view, determining teachers' present understandings about teaching and learning becomes important.

Alkan, Köroğlu and Başer (1999) noted that there have been certain weaknesses generally speaking, in our educational system and particularly in training mathematics teachers. And he discoursed that on this stage, we should stay up to date about the changes and relieves the weaknesses. Related to teacher training, Baki (2008) highlighted the reason why many educational improvement projects and innovations have failed, was undermining the important role of teachers during these attempts. He claimed that the significant changes that are expected to occur in school mathematics can only be achieved when there is a distinguishable improvement in teachers' attitudes towards mathematics and thoughts about mathematics and its instruction.

As it can be concluded from the explanations above; first, teacher beliefs, which shape the classroom applications should be determined in order to change traditional classroom applications, and to implement more effective instructional applications with more constructivist notion.

There were lots of studies investigating teacher and candidate teachers' beliefs in mathematics education. Nevertheless, there were not enough studies on classifying teacher beliefs with respect to certain theoretical framework. Additionally for Turkish context, as the base of teachers' accustomed to mathematics teaching approaches, present beliefs of mathematics teachers about learning mathematics and nature of mathematics knowledge should be investigated in detail in order to achieve application of the new curricula successfully. This study aims to present a cross-section of opinions and thoughts of the mathematics teachers about mathematics knowledge, learning-teaching and assessing mathematics. With

the framework of the aim above, this study sought solutions for the following problems:

1) What are the opinions of participant about the nature of mathematics knowledge?

2) What are the opinions of participant about the teacher's role in teaching mathematics?

3) What are the opinions of participant about the assessment?

Method

In this study, the case study method was adopted in order to give better respond to research problem. Case study method is selected because of it is particularly suitable for individually implemented studies and enables an in-depth inquiry of one aspect of the problem, and can be completed in a shorter time frame (Çepni, 2007). This study was conducted in the fall term of 2008-2009 instruction year, with a 5-year experienced mathematics teacher working for a secondary school affiliated to Trabzon province Directorate of Ministry of Education.

1. Data Gathering Tools and Procedure

An interview form prepared was used as the data collecting tool included open-ended and scenario type questions by the researcher. In addition, a form with photos visualizing different classroom environments in was used to gather more detailed data and to incite the interviewee to discuss. The interviewee was interviewed four times, each of which took average 45 minutes. The interviews were first recorded with a digital sound recorder and then transcribed.

2. Data Analysis

The obtained data were analyzed descriptively on the basis of Magolda's Epistemological Reflection Model. From simple to higher levels, the perceptions in this model were ranked as; Absolute Knowing, Transitional Knowing, Independent Knowing, and Contextual Knowing. The dimensions of Baxter Magolda's (1992) Epitemological Reflection Model have been summarized by Collins (2005, p.58) under four levels as displayed in Table 1.

Table 1 Epistemolojical Reflection Model of Magolda

Dimensions	**Level 1 Absolute Knowers**	**Level 2 Transitional Knowers**	**Level 3 Independent Knowers**	**Level 4 Contextual Knowers**
Role of students	Acquiring the knowledge from teacher	Comprehending the knowledge	Self-exploration, constructing, developing, and sharing own opinion with others	Formatting problem situation, making opinion transfer and contrasting them
Role of peers	Sharing teaching materials, explaining the learning to their peers	Taking active role in knowledge exchange	Sharing opinions and serving as a valid information source	Helping to learning through quality contributions
Role of teachers	Presenting the knowledge truly, make the students understand the issue	Using appropriate methods that ease the comprehension, and help students to apply what they have learned	Supporting independent thinking of students, promoting them to share their learning	Helping to apply knowledge with respect to the context, promoting students to discourse from the critical point of view, and student and teacher criticize each other
Evaluation	Evaluation is a tool displays to teacher what has been learned	Indicates to what extend an issue acquired by the students	Rewards the independent thinking	Measures the skills truly. Student and teacher work together with respect to targets and measure the development together

Nature of knowledge	Knowledge is definite and absolute	Some parts of the knowledge is absolute, while the some parts are not	There is not any absolute knowledge. So, everyone can defend the integrity of own opinion	The integrity of the knowledge is contextual. Each knowledge can be evaluated according to its conditions

Findings and Interpretation

The findings were analyzed and presented under three titles concerning the problems of the study.

1. Participant's Beliefs about the Nature of Mathematics Knowledge

The participant's beliefs about the existence and certainty of mathematical knowledge were investigated under this title. The views of the participanton the issue are below:

"… Mathematics has ever existed in nature. We live it in life anyway. I guess, one of our teachers at university said that we obtain the natural numbers from Peano's axioms. All numbers form out of "1" there. First, the existence of "1" was assumed and then all numbers are produced consecutively. So, there is mathematics in the nature. I don't think that we produced mathematics. I think, we research and discover things that already exist…"

As is seen, the participant described the mathematical knowledge as the discovery of factual knowledge existing in nature instead of defining it as an output of human cognition. The participant also gave the following explanation referring the certainty of the results of mathematical problems:

"… Result is certain in mathematics but there is variety of ways to reach the result… The difference between these ways comes from power of interpretation. You can reach the result via different means by interpreting the problem differently…"

Fritz, Robyn, Chasity, Melisa, and Fred, participant, in the study of Weinstein (1998), gave some parallel expressions with our participant in this study. As an example, Fritz's opinions about nature of mathematics knowledge are briefly presented below.

Fritz: *"Most problems have one true answer… I think you just need to know how to come up with a real answer, the true… It's always going to be different information, but one way or another, you can always use a procedure to come up with the right answer… and you can apply logic if you can't quite figure out a problem in finite math. There's definitely more than one way to come up with the right answer, and you can think about it"* (Weinstein, 1998, p. 84-85).

In another expression, the participant consolidated his belief to certainty and clarity of the mathematical knowledge by comparing it with literature.

"… Literature course is an infinite science… because it is open to interpretation and is a science performed by feelings. Writing poems, telling stories, interpreting a piece of literature are rather done by feelings. You produce something. Whereas mathematics is explicit…"

As understood from the expressions above, the participant defined mathematical knowledge as certain and explicit. On the other hand, he perceived literature as a science open to interpretation and human production. The dualist point of view of the participant between certainity and uncertainity, open to interpretation-not open to interpretation, and production of mankind-not production of mankind turned into absolutist one while considering the mathematics. The participant put mathematical knowledge in the category where the knowledge is characterized as certain, indisputable and not overt to interpretation, in this dualist point of view. The participant's dualist point of view was similar to Alice, Megan, Carl and Al's expressions in Magolda's (1992) study. As an example, Alice's opinions about this issue are briefly presented below.

Alice*: " I've noticed that a lot of things professors say are opinions, and it's their own experience. It has to be because it's not real factual material. If it is a definite fact or statistic or a definition, they'll say. But in general lecture, you can tell that it's from their personal experience. I think that is good in a lot of ways, though, it makes me think, " what I ever done that fits into this?" If one made more sense to me personally, that's probably the way I would go with it. Just take it on a personal basis, realy…"*(Magolda, 1992, p. 115).

The similar expressions were also discoursed by a participant defined on the 2nd level in the study of Belenky, Chincy, Goldberger, and Tarule (1986) and another participant in dualist level in Perry (1970)'s work.

"There are absolutes in math and sciences. You feel that you can accomplish something by-by getting something down pat. Work in other courses seems to accomplish noting, just seems so worthless. It doesn't really matter whether you are right or wrong, ' cause there really isn't a right or wrong…" (Belenky et al.,1986).

"… I'll tell you the best thing about science courses: their lectures are all right. They're sort of, they say fact. But when you get to a humanities course, especially- oh, they're awful!- those lectures. Oh, I can't see any relation. You're reading a book, and, ah, to my way of thinking, anyway, the lecturer is just reading things into it that were never meant to be there. They say that, ah, I can't see how they can draw a conclusion…" (Perry, 1970, p. 79).

The participant repeatedly remarked that mathematics knowledge is certain (fixed truths) and it is developmental at the same time. He attributed this to the existence of unsolved problems in mathematics and knowledge which has not been discovered yet in nature.

As a summary, according to the participant, mathematical knowledge is not a product of the human mind but the discovery and replica of the factual knowledge in nature. So, mathematical knowledge is composed of facts that are not open to interpretation, certain, and explicit (fixed truth). It can not be discussed and falsified. Indeed, mathematical knowledge is developmental. The reason for that is there are still unsolved problems in mathematics and there is undiscovered knowledge in nature. The developmental nature of mathematics is finite. The only discussible issue in mathematics is there are unsolved problems in mathematics. Each problem has a unique solution but solution ways differ. The problems which will be solved in the future will also have unique solutions.

As a result, there can be said for the participant having absolute knowing level view in terms of the nature of mathematical knowledge since he frequently described mathematical knowledge as existing in nature, certain (fixed) and explicit, not including contradictions (having inner consistency), can not be interpreted in a different way, can not be discussed and falsified by time. The opinions of the participant on nature of mathematical knowledge were also summarized in Figure 1 (See Appendix A).

2.Participant's Beliefs about the Teacher's Role in Teaching Mathematics

Under this title the answers for the following questions were sought*:* "Is mathematics teacher the only source of information? Or, can students learn mathematics by themselves?" And related to this main question, the opinions of the participant about effective teacher, instructional methods and techniques he uses, the ideal classroom environment he adopts, instructional materials he uses and technology usage particularly in mathematics instruction were discussed in detail.

2.1. Who is The Effective Teacher?

The participant's opinions relating to the effective teacher are as follows:

"… A saying tells: "If you tell me, I forget. If you show me, I may remember but if you make me join I understand and do…" I think student participation is a very important factor. And, appealing to level of students is another one. I mean concerning the student level in each classroom is important… Provide student participation. What else? Trying to make them like the course..."

It was observed that in this part of interview the participant defined a teacher type who tries to make students participate to lesson, to get on well with them, to make them like the course by making them love him, to prioritize appealing to students' level. Additionally the expression, *"… and appealing to level of students…"* by the participant shows that he concerns the pre-knowledge and present perceptions of students.

It was concluded that the participant also considers about individual differences and he tries to use methods that facilitate the lesson to be understood. His opinions about that are below:

"…We instruct by considering variety of learning styles, in fact. Actually, some students learn by explanation but some others don't understand by lecturing without joining lesson. And still some others are the students that they can not learn without doing by themselves. I mean this is multiple instruction theory. We are taking this into consideration anyway…"

As is seen with the expression, *"…We instruct by considering variety of learning styles, in fact."* the participant repeated one of the characteristic of transitional knowledge level that is about *"teacher instruction with different methods to make students understand the lesson better."*

2.2. Learning Environment

The participant's opinions relating to the learning environment are as follows:

"… In the present situation students have a classroom but we are thinking teachers to have their classrooms. For example, I will have a classroom. The classroom of mathematics teacher…I will equip that classroom with all tools related to mathematics. Everything that I need will be there. Computer, data projector etc…"

The participant apparently adopted computer assisted classroom environment. But as seen below, the participant took technology as a part of traditional instruction based on presentation-receiving and as only a presentation tool in teacher's hand.

"… We have computer and data projector in technology classroom. We can explain certain things to students by projecting on the board… When we make it fun and include technology, students naturally will get a bit more enthusiastic… Also in the computer lab, we have a computer for each student… We can utilize these opportunities in this manner…"

Based on *"…We can explain certain things to students by projecting on the board… When we make it fun and include technology, students naturally will get a bit more enthusiastic…"* expressions by the participant, we understand that he considers educational technology as a presentation tools which makes lesson fun, enriches education rather than a learning tool in students' hand which enable them to perform individual and group works, discover by researching and investigation. By discoursing *"If there is both technology and groupwork, it will be even a higher quality of education anyway",* he remarked that he accepts classrooms that are equipped with computers and suitable for groupwork as ideal classroom environments. Concerning the teacher roles, these perceptions are at subjective knowing level of Belenky et al (1986), transitional knowledge level of Magolda (1970) and pluralism level of Perry (1970).

The participant, who emphasized that technology is more practical in courses like history or geography etc. to make the lesson more interesting than mathematics and science courses, parallel to his teacher centered instruction understanding, also perceived technology as a presentation tool in teacher's hand. This situation may be a result of his belief about mathematics knowledge exists in nature, independent from individuals, so it can directly be transferred. When it was reviewed, the participant had never mentioned that computer and other instruction materials could be a learning tool in students' hand.

2.3. Teaching Techniques

The participant's opinions relating to the teaching techniques are as follows:

"…Questioning is the most frequently used method by us. I don't much prefer direct lecturing… We try to make students participate with questioning. The most frequently used method by us is that…"

The participant summarized a typical lesson as follow:

"… I will explain what I do at least. First of all, if I am starting a new unit, I talk with my students about the related issue in the first place… As: "We will cover these." If we can use it in daily life, I talk about these applications. If we can not use it in daily life I mention about the importance of the issue in the National Student Selection Examination (ÖSS). Students should know why they are learning that topic. I prefer to start with questions before I start a unit… Let's say, if we are at 'sets' unit I ask "What do you think the set is?" For example, "Give me an example for sets.", "Why have you said that?", "Is that alright?", "What if we did like that there?", and so on… This makes following steps easier…The teacher will show the way and students will follow him in somehow… The student participation is extremely important…"

It is obvious that the participant generally use questioning and lecturing methods. However, the

participant himself reported that he does not use lecturing since it is not an effective teaching method. The participant, who seemed to confuse questioning with lecturing, rationalize he is not able to teach based on discovery learning with heavy load of the curriculum, time restrictions, and expectations of the society.

The participant defined a rather teacher-centered instruction, he gave little emphasize on students can learn by discovering and he adopted the teacher as the source of information. It can be concluded, that he had a perception lower than independent knowing level, from his interpretations on in class activities. As it is going to be mentioned in the parts related to assessment in this article, the participant noted that he applies periodical monitoring tests at the end of each week or unit both to render his students achievement in ÖSS and to monitor how much the students understand the unit. The participant said that the present circumstances push teachers towards applications based on memorization of knowledge and solving lots of test items for the sake of student's achievement in ÖSS. He also added that the success of teachers also being measured by the parents, students, and other educational autorities through how many students he/she made university student. He underlined that actually he is not satisfied with such applications, which promises that he could be open for innovations if the expectations change in a better way.

2.4. Teaching Materials

The participant's opinions relating to the teaching materials are as follows:

"… We have no problems in terms of material. Actually, we don't use many materials anyway. What can we use in mathematics? May be sometimes ruler and protractor. We can use them while drawing on the board. We do it gropingly anyway…"

The participant reported that except for ruler and protractor he does not need and use any of the instructional materials very much. Moreover, it seems that he draws figures gropingly to save time and he uses data projector as a presentation which means only to cheer up the lesson occasionally. These views of the participants partially contradicted to his expressions above about using technology. Above, the participant put forward a view favoring technology and mentioned that each course should have in its classroom. This contradiction in his views may come from that he had no special training about how to apply technology in mathematics instruction and his lack of detailed information on conditions that he can use technology in mathematics instruction for a specific aim. The participant's repeating the expressions such as *"… as far as I know, as I heard from the colleagues of mine who use it…"* about the smart-boards supports this idea.

Concerning the views above related to teacher role, it can be said that the participant concerns

students' pre-knowledge, present perceptions and individual differences. He values comprehending rather than memorizing. He generally instructs his lessons based on questioning and lecturing techniques. Although he is not against technology, he takes instructional technologies like computer and data projector as presentation means in teacher's hand to make lessons fun and enrich instruction. These perceptions belong to "Transitional Knowing Level", in which the knowledge is certain, the teacher is regarded as the unique source of authority and comprehending is prioritized. Therefore, it can be concluded that the participant's opinions about the teacher role accumulated on transitional knowledge level perceptions.

3. Participant's Beliefs About Assessment

Under this heading, the participant's ideas about what kind of measurement tools the participant uses for evaluation (how he measures student achievements) and how they conduct assessment were discussed elaborately.

The participant's opinions relating to the measuring tools are as follows:

"I hand over related tests to students while I am instructing the unit anyway. And at the end of the unit, I distribute a test related to that unit, we call monitoring test… Here subjective evaluation is carried out in that way. And we have essay examinations. I, of course, should train students to solve multiple choice tests for ÖSS preparation… But I believe that essay exam is a must in order to decide if a student has understood the unit or not… I should see students' solution strategies rather than the result. I should see how they came up with that solution, which strategies they used…"

As understood from the expressions above, the participant uses monitoring tests as he named as, "subjective measurement" to consolidate the unit he has finished. The participant mentioned that he prioritizes how students interpret and solve questions, the necessity of essay examinations rather than multiple choice exams carrying the risk of accidentally high grades by random choosing. However the expressions of the participant below show that in his measurement he scores whether students find the right answer instead of a grading system valuing the procedure in solving questions. When the participant was asked: "Suppose that you asked a question like solve the: "2x+4=7" equation and some of your students solved the question as "2x+4=7 = 2x+4-4=7-4 = 2x = 3= x = 3/2". How many points would you score to those students?" He said he might give 9 out of 10.

As noticed, against the explanations above, the participant noted that he could give 9 out of 10 only for right result reached. The participant said that procedure and thought in each step were right, and added that "=" symbol was consecutively used in a wrong way. It can be said that the participant, who

said he might give 9 out of 10, put emphasis on the result rather than the procedure. The reasons that push the participant to give contradicting expressions may be on one hand he gives importance on comprehension but on the other hand the present examination system and examination related expectations of the society push him in the opposite direction. About this issue the participant noted the things below repeatedly:

"… As I said, what are the criteria of our success? What are their final assessment measures about us? They evaluate if our students are successful or not according to ÖSS. So, the thing we should do is make students get fully ready for ÖSS at the right time. Why parents send their children to private schools? To learn mathematics persistently? No. They send so that the child could solve all mathematics questions in ÖSS…"

Based on the explanations above, it was understood that the participant assigns long term research works, applies essay examinations, and unit monitoring tests. In addition, he remarked that he supports student grade averages with teacher opinion based grades. The participant did not mention alternative assessment methods such as self-assessment, peer assessment, and portfolio. Still, the explanations above show that he overts to innovative applications, he already knows about some of them but not in detail, and he does not use some applications because of the reasons mentioned above (ÖSS system, expectations of the society from teachers). The participant seemed to value whether students reach the right solution in questions. It can be seen that ÖSS system, present level of students, and the expectations of the society (especially guardians') affects the beliefs of the teacher related to assessment. The participant gives importance not only for the multiple choice tests but also the questions call for comprehending and interpreting. However, he prioritizes solving multiple choices test for the sake of students' ÖSS achievement. The participant is open to innovations in terms of assessment but is not informed enough about the issue. As a result, it can be said that the assessment related views of the participant condensed ontransitional knowledge level but he is readily open to the views above this level.

Discussion and Conclusion

Supported with the findings above, it was concluded that the participant was at absolute knowing level in terms of nature of mathematical knowledge and at transitional knowing level in terms of teacher role and assessment. As Baki (2008), and Baydar and Bulut (2002) noted, the teacher perceptions about the nature of knowledge affect their learning-teaching and assessment related approaches. From this point of view, it is meaningful that the participant had close levels of view about mathematical knowledge and perceptions about teacher role and assessment. Besides, even though the participant does not have detailed knowledge about especially on teacher role and assessment, he is open

to new applications. This shows the necessity of active and continuous inservice trainings where teachers can gain knowledge and experience. Moreover, concerning that teachers' beliefs related to nature and instruction of mathematics affect the methods they use, and so student beliefs are indirectly affected by them (Skott, 2001), which further escalades the importance of such in-service trainings.

Today, the role of beliefs in mathematics education is undisputable anymore. The researches over teacher beliefs have been increasing in number and getting deeper. Beliefs related to mathematics education, their effects on education, how they affect daily life of teachers must be investigated with the framework of present educational reform initiative in our country and prospective studies to improve the quality of mathematics education. Teacher beliefs about nature and instruction of mathematics, is an issue that one who would like to understand what is happening in the classroom should not skip. For this reason, we think that carrying out similar studies with larger samples in order to have more detailed and various information and using different intellectual models like Belenky et al (1986) and Perry (1970) collectively can contribute to educational reform studies in the agenda of Turkey.

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Appendix A

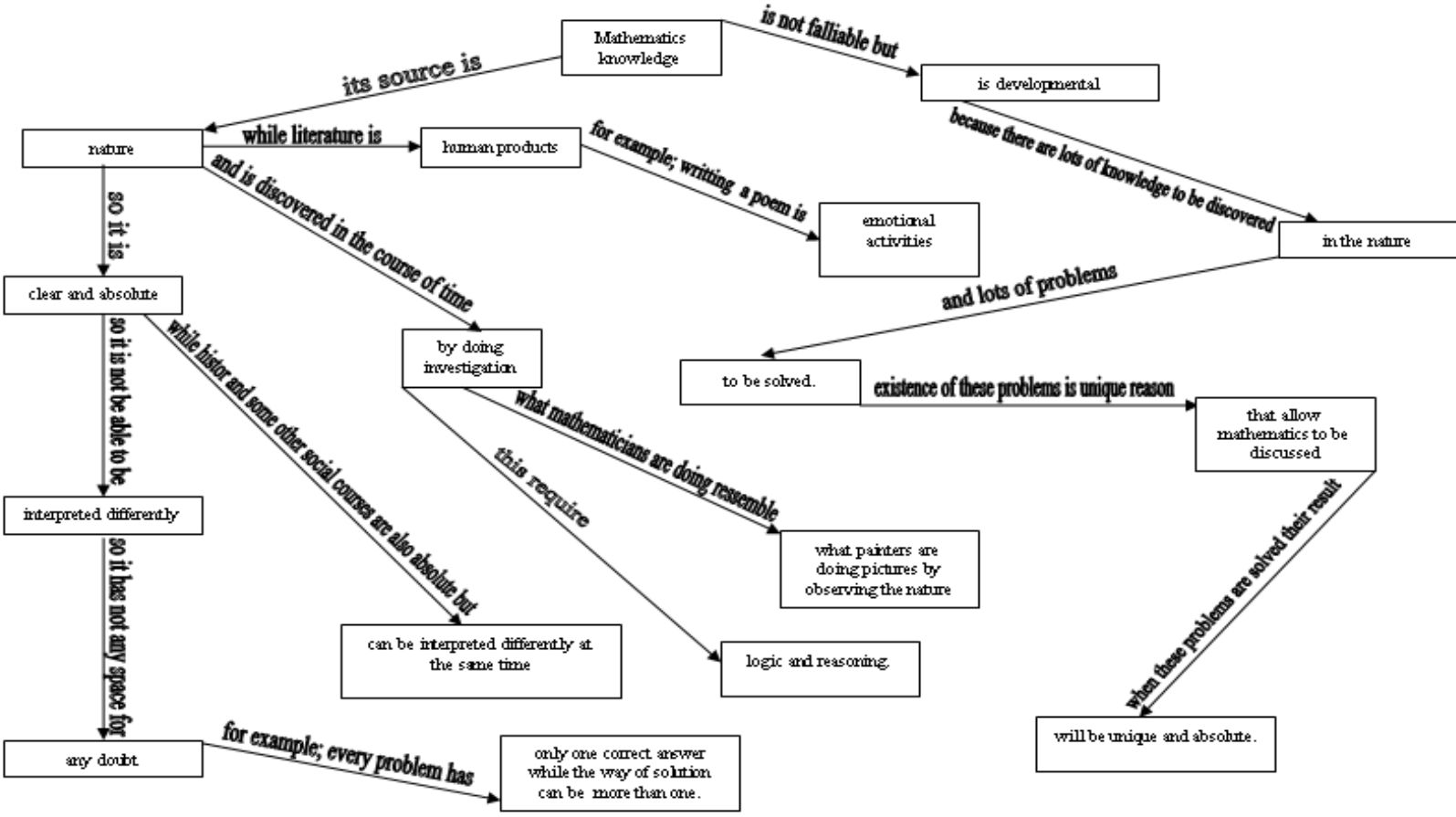

Figure1. Participant's cognitive map about the nature of mathematics