



<http://www.eab.org.tr>

Educational Research Association  
The International Journal of Research in Teacher Education  
2019, 10(4): 68-80  
ISSN: 1308-951X



<http://ijrte.eab.org.tr>

## Students' Reasoning and Utilization of Argumentation Skills in Solving Chemical Kinematics Calculus-Based Problems

Paul IWUANYANWU<sup>1</sup>

---

### Abstract

Developing students' reasoning and utilization of argumentation skills in chemical kinematics entails learning to use basic facility of derivatives and integrals and their applications effectively and efficiently as applied to the context of undergraduate general chemistry course. It has been necessary to provide the students of this study with empowering learning experiences, helping them to develop both thinking and reasoning skills for use in solving chemical kinematics calculus-based problems. The study sample was 66 (31 males, 35 females) undergraduate second year chemistry students taken from a population of 123 full-time registered students in interdisciplinary subject areas in chemistry, physics and biology. Participants received their learning of chemistry via argumentation instruction for 14 weeks during which data were collected. A cross-case analysis was followed to interpret character of reasoning and arguments students generated through activities in chemical kinematics. Results indicated that students who successfully solved the task were engaged in analytical thinking and creative reasoning and used substances of arguments extensively. In particular, this suggested that utilizing argumentation skills for solving chemical kinematics calculus-based problems means framing predictive and verificative arguments that support the solution. Implications of the findings are discussed.

**Keywords:** Argumentation instruction, reasoning, thinking, calculus, chemical kinematics problem-solving

---

**Received:**02 April 2019    **Accepted:**29 September 2019    **Publish:**30 December 2019

---

<sup>1</sup>Lecturer, Science Education , University of the Western Cape, Cape Town, South Africa

**Correspondence:** piwuanyanwu@uwc.ac.za, ORCID ID: 0000-0001-7641-6238

## Introduction

Learning is a consequence of reasoning and thinking (Perkins, 1992). Thus, the traditional view that the basics of science can be taught as routine skills, with thinking and reasoning to follow later, can no longer guide the science education instructional practices (Walker & Sampson, 2013). In fact, developing students' reasoning in science classroom is no longer viewed as an optional activity that students may or may not get to at the final stages of learning a new concept (Zohar & Nemet, 2002). Instead, reasoning and thinking are applied to all learning. In science teaching, as in chemistry, students' reasoning and knowledge resources, are important because doing and learning chemistry involves a wide range of scientific practices in constructing evidence-based knowledge claims and solutions, such as asking questions and defining problems (Patel, 2012; Walker & Sampson, 2013). The foregoing also include developing and using models, planning and carrying out investigations, analyzing and interpreting data, constructing explanations, engaging in empirical arguments, and evaluating, and communicating information (Bain & Towns, 2016). Going by this then, it is unlikely that a student will excel in general chemistry course without a solid understanding of and facility with the knowledge and skills in the areas of basic mathematics, calculus, and 3D geometry. And, whilst all these factors should be considered in teaching and learning of chemistry, several studies have shown that students main reasons of difficulties in developing them are due to lack of making logical connection between symbolic and embodied notations rooted in the structure of chemistry content or physics content for that matter (Azarang, 2012; Carius, Júnior, & Silva, 2017; Hashemi, Abu, Kashefi, & Rahimi, 2014; Kurt & Ayas, 2012).

### The Use of Calculus in Chemistry

Physical chemistry courses often have contents in single variable calculus, multivariable calculus, and differential equations with modeling (including other mathematical contents such as linear algebra, and statistics i.e. data analysis). Organic chemistry includes 3-D and would require experience in multivariable calculus (or somewhat restructured calculus II course). Thus, students taking chemistry would benefit from experience of having taken courses related to calculus. It can be assumed that the learned skills and knowledge gained from taken calculus course prior entering or during undergraduate chemistry course would serve students better in their subsequent dealing with calculus-based chemistry problems. For example, in deriving a calculus-based chemical kinematic equation in which two different kinds of molecules X and Y react to give products. In this example the rate is likely to be given by a second-order equation of the form  $v = \frac{dp}{dt} = kxy = k(x_0 - p)(y_0 - p)$  in which  $k$  is now a second-order rate constant.

The integration can be achieved by separating the two variables  $p$  and  $t$  in this form

$$\int \frac{dp}{(x_0 - p)(y_0 - p)} = \int k dt .$$
 For chemistry students with limited mathematical knowledge,

the simplest and most reliable method for integrating the left-hand side of the equation is to multiply both sides of the equation by  $(y_0 - x_0)$  and separating the left-hand side

into two simple integrals:  $\int \frac{dp}{x_0 - p} - \int \frac{dp}{y_0 - p} = \int (y_0 - x_0)k dt .$  Hence,

$\ln(x_0 - p) + \ln(y_0 - p) = (y_0 - x_0)kt + \alpha .$  That being so, putting  $p = 0$  when  $t = 0$  we find

$\alpha = \ln(y_0/x_0)$ , and so  $\frac{x_0(y_0 - p)}{y_0(x_0 - p)} = e^{(y_0 - x_0)kt}$ . Interestingly, a special case of this result is

important, for example, if  $x_0$  is negligible compared with  $y_0$ , then  $(y_0 - x_0) \approx y_0$ ,  $p$  can never exceed  $x_0$ , on account of the stoichiometry of the reaction, and so  $(y_0 - p) \approx y_0$ . And of course, by introducing approximation  $\frac{x_0 y_0}{y_0(x_0 - p)} = e^{k y_0 t}$ , we can derive a pseudo-first reaction, where  $k y_0$  is a pseudo-first-order rate constant, and remembering that  $1/e^{k y_0 t} \equiv e^{-k y_0 t}$ , this can then be rearranged to read  $p = x_0(1 - e^{-k y_0 t})$ . In this sense, what has been inescapable in studying chemical kinematics is the impact that certain units of calculus have on what we teach given the fact that derivatives can be applied to find the maximum and minimum component of chemical variables. Whilst, calculus use in chemistry and physics has been successful because of its extraordinary power to reduce complicated problems to simple rules and procedures (e.g. Bain & Towns, 2016; Festus & Ekpete, 2012; Patel, 2012; Zhdanova, Kuznetsov, Legros, & Strizhak, 2017), it seems this is not the case with students of all ages from all walks of life. Many of us who teach calculus use in undergraduate science courses do not at present have experienced success in our students learning of calculus use in chemistry, for example, in chemical kinematics.

An overriding difficulty, and therefore one that can lead to the greatest excellence if confronted and resolved, is finding a suitable instructional strategy that can help to tackle the problem of the continued poor performance of undergraduate year 2 chemistry students involved in this study. Some of the difficulties peculiar to chemistry that students tend to encounter are self-inflicted; others stem from specific features of the discipline. A more recent study done by Carius et al. (2017) revealed that their undergraduate chemistry students credit their difficulties in making conceptual connection between mathematics (differential calculus) and chemistry. As part of their remedial instruction, they utilized alternative instruction (mathematical modeling) to relate mathematics and chemistry in a discipline of differential and integral calculus. What serves to orientate the focus on these difficulties is the specific close association chemistry has with mathematics. It is possible to teach the subject as nothing but the rules and procedures-hereby losing sight of both the mathematics and of its practical value in chemistry. The point here is that adequate knowledge resources in the form of concepts, laws and theories cannot often be made in the means without evidence of a strictly scientific processes. Processes by which knowledge evolves inductive and deductive techniques needed to integrate various levels of algorithms that lead to calculus-based chemistry solutions.

Despite the rich literature on differential calculus and chemistry, there is still room for more work to be done in finding new ways to help students to better their understanding of chemistry. Therefore, insights into how different engagement in reasoning and utilization of argumentation skills among students relate to successful or unsuccessful solving of chemical kinematics calculus-based problems could be important in the development of learning instruction. By this token, the research question guiding this study is:

How do students' reasoning and utilization of argumentation skills relate to their success in solving chemical kinematics calculus-based problems?

To explain the rationale for remedial instruction that integrates explicit teaching of knowledge and reasoning patterns with calculus-based chemistry content, it is necessary to review some of the literature about the relationships between knowledge resources

and reasoning.

### **Theoretical Background**

Previous studies have shown that students of all ages have considerable difficulties in developing reasoning and thinking skills needed to solve calculus-based problems (Azarang, 2012; Hashemi et al., 2014; Lithner, 2008). Some studies, however, have shown that difficulties in reasoning reflect superficial mental models of situations rather than logical fallacies (Kapon & diSessa, 2012; Walker & Sampson, 2013; Zohar & Nemet, 2002). When studying how students learn chemistry, certain cognitive principles have to be considered. According to diSessa (2004), there are many different levels at which a concept can be understood, and contextuality has to be taken into consideration. In other words, to understand a student's learning difficulty of a particular concept, his or her understanding has to be studied in a variety of different contexts. Researchers, Bing and Redish (2009), Festus and Ekpete (2012), and Jonassen (2007) mentioned how difficulties in learning physics (also applicable in chemistry) not only stem from the complexity of the subject but also from insufficient mathematical knowledge that students hold. Since mathematics is a natural part of chemistry, it is reasonable to assume that the ability to use creative mathematical reasoning is an integral part of the chemistry knowledge students are assumed to achieve in chemistry courses.

Creative mathematical reasoning (CMR) is characterized by three important constructs, namely, novelty, plausible argumentation and mathematical foundation. These constructs are used as an extension of a strict "mathematical proof" to justify solution and are seen as a product of separate reasoning sequences (Lithner, 2008, p.266). Each sequence includes a choice of algorithm that a student made to define the next sequence, and the reasoning is the justification for the choice that is made. In any case when a student's reasoning constitute memorized facts such as an approximate value of proton rest mass  $m_p = 938.3 \text{ MeV}/c^2$ , algorithms or procedures for how to solve a problem; it is considered as imitative reasoning (IR). Thus, IR such as algorithmic reasoning (AR) concerns the application of provided or memorized algorithms to solve a problem. When students are operating at this level they not only define the choice of the algorithm that they use, they are compelled to generate substantial arguments to buttress their solution pathway. Depending on the argumentation for the choice of the used algorithm, the reason for the strategy choice can be integrated into other arguments supporting the strategy and the available prior knowledge base in a content domain.

The effect of prior knowledge on argumentative reasoning is another important variable that has been studied. For example, knowledge is found to be related to some aspects of argumentative thinking, such as generating more reasons or stating more qualifiers, but not to other aspects (Means and Voss, 1996). The arguments for the chosen solution method (i.e. reasoning) can be anchored by the following criteria: a) "novelty – a new reasoning sequence is created or a forgotten one is recreated", b) plausibility – students generate arguments that support the strategy choice and or strategy implementation motivating why the conclusions are true or plausible, and c) mathematical foundation – the arguments made during the reasoning process which are anchored in the intrinsic calculus properties of the chemistry components involved in the reasoning (Lithner, 2008, p.266).

Furthermore, augmentation may be considered predictive – that is to say, a mathematically anchored justification for why a strategy will work – or "verificative" – i.e., a mathematically based explanation for why the solution worked or did not work

(Lithner, 2008, p.263; Voss, 2006). Moreover, development of these fundamental skills aligns with analytic, dialectic and rhetoric nature of argumentation skills. It aligns with analytic argument, which is grounded in the theory of logic and proceeds inductively or deductively from a set of premises to a conclusion. For example, a chemistry educator may ask, 'what does this reaction tell us about the substances we mixed in the test-tube and why is a gas given off? Finding answers to this question subsume a number of basic processes, skills, and abilities (e.g. observation, measurement, inference, prediction, classification, controlling variables, scientific language, critiquing solutions and explanations, using logic, etc.). It also aligns with dialectic argument, which occurs during discussion or debate, and rhetoric, which is employed to persuade an audience (Toulmin, 2003). With this, pedagogical matters relating to approaches in developing students' reasoning and argumentation skills for use in solving calculus-based chemistry problems require compartmentalized endeavor or a series of skills. It is also necessary to note the reference to other attempts in which argumentation was used; both as an instructional approach to teaching science problem-solving to students at different levels of learning, and as a way of promoting meaningful learning (See for example, Author, 2019; Berland & McNeill, 2010; Carius et al., 2017; Diwu & Ogunniyi, 2012).

### **Methodology**

In order to achieve the aim of the present study, a case study research design commonly used in similar studies that examine research participants' reasoning, thinking, arguments, problem solving skills, or views was employed (Frankel & Wallen, 2009). For example, Bing and Redish (2009) used a case study to analyse mathematics use in physics problem solving among undergraduate students with the intention to establish evidence of their epistemological framing via warrant. In the present study the participants were 66 (31 males, 35 females) undergraduate second year chemistry students taken from a population of 123 full-time registered students in interdisciplinary subject areas in chemistry, physics and biology. Participants were ranged between the ages of 19 and 23. They came from working class, dual income, and middle-class income families who are residence in rural, pre-urban and urban areas.

### **Procedure**

The length of the study was approximately 14 weeks. Twelve weeks of the study were devoted to teaching integrated calculus-chemistry concepts and the 13<sup>th</sup> week was devoted to addressing students' inadequacies and providing feedback. Classes were held 2 days a week (Tuesday and Thursday) for 60min per class session. Students who were at risk of failing the course were encouraged to participate in a complementary practicum which met once a week. The objective of the practicum was to motivate the students, mentor them, and use the conceptual resources from the items addressed in weeks 1 – 12 to strengthen their mathematical reasoning. Utilizing the insights from prior work in chemical kinematics which took into consideration students' collective prior knowledge, abilities, problem-solving skills, reasoning and thinking skills and experience, ten problems were constructed (Bain & Towns, 2016; Kurt & Ayas, 2012; Patel, 2012). The items dealt with problems in: a) integrated rate laws for more than one reactant, first-order, half-life, second-order, zero-order; b) reaction mechanism, and c) a model for chemical kinematics. The purpose of the items were for students to: a) experience and use their learned knowledge and skills from solving chemical kinematics calculus-based problems to better their understanding in other related areas of science, and b) enrich their chemistry and mathematics content knowledge. The ten items were



reviewed by two experts who have more than 22 years teaching experiences in chemistry and mathematics. They were asked to critique the items as to whether they can measure the specific reasoning and thinking skills and to suggest any necessary modifications. Suggestions were made; corrections were affected to the items. After this data were collected (60 min in week 14). Students were asked to participate in ten chemical kinematics calculus-based problem solving session.

In practice, the instructor presented the challenging problems to the students without the solutions, the problems created some form of cognitive dissonance, and the students engaged in a productive struggle to solve the problems. Students learning instruction was informed by the following authors' views on argumentation in the extant literature (Diwu & Ogunniyi; Lithner, 2008; Toulmin, 2003). In line with the nature of the study, argumentation practice was implemented as follow: Intra-argumentation i.e. the brainstorming or self-conversation stage. At this first stage, students performed individual tasks at least one time in each of the ten calculus-based chemistry problems. In situations in which students became stuck and cannot proceed further, the instructor encouraged them to explain their ideas and strategies to help them move on in their task. At any rate when students had solved given problems (or had given up), they were invited to explain why they thought that their solution pathways had or had not been successful. Upon completion of individual task, students were asked to work in small groups (inter-argumentation) and complete the task. At this point each small group of 5-6 students received tasks that required the transfer of knowledge resources of chemical kinematics calculus-based problems to different contexts which required different levels or types of arguments. During this time, the instructor moved among the groups, probed the students' ideas, and asked for explanations to the reasoning used. After this stage, students were asked to place their worksheet solutions on the document camera which projected to an interactive whiteboard. Next, they were asked to be prepared to present their worksheet solutions and justify their proposed solutions to the class. During this process, other students were invited to identify instances of valid and invalid arguments used by the presenters to support their problem solution pathways. Mistakes were identified, and corrected. At the conclusion, trans-argumentation or whole-group discussion and reflection, collaborative consensus were reached on solutions that students worked out. Students' activities were videotaped, with integrated voice and video recording. Some of the typical responses to the chemical kinematics calculus-based problems have been presented in the results section of this study.

### **Data Analysis**

The research question concerns how students' reasoning and utilization of argumentation skills relate to their success in solving chemical kinematics calculus-based problems. Students' reasoning was categorized using Lithner's (2008) framework of creative mathematical reasoning and imitative reasoning (CMR and IR). How students used argumentation skills in which analytic, dialectic and rhetoric forms of arguments shape their learning of chemical kinematics and solving of problems involving the concepts were examined using the concepts of predictive and verificative argumentation (Lithner, 2008; Voss, 2006). The students' dialogues in each phase of the ten items were examined, and units of argumentation, that is to say, the implicit and explicit justifications of the strategy choices were identified. For example, students' reasoning was considered CMR if there were instances of creating plausible solution method (that may contain some elements of algorithmic reasoning). On the other hand, students' reasoning was categorized as IR if the task solution pathways were based

essentially on familiar facts and /or procedures only. Thereafter the relationships between students' CMR and IR and predictive and verificative arguments were analyzed considering whether important decisions in the problem solving process were consequences of certain reasoning and the utilization of argumentation skills. To achieve this, dialogues (including negotiating meaning of concepts and sharing tacit understanding) during phases (verificative and predictive) before and after each activity is completed were noted. What they said in planning how to proceed with the solving of a given problem (predictive arguments) were considered in the context whether the reasoning was associated with the characteristics of CMR or IR (verificative arguments).

### Findings and Discussion

The first result based on students' reasoning and utilization of argumentation skills came from a group of five students who were dealing with item #2 of the chemical kinematics calculus-based problems. The problem statement reads:

**Item #2:** *Based on time dependence of concentration, use the integrated rate laws (average rate and instantaneous rate) to determine whether the concentration of a reactant governed by the first order kinematics falls off from an initial concentration exponentially with time.*

During this episode, the students are trying to decide if the concentration of a reactant governed by the first order kinematics falls off from an initial concentration exponentially with time. They negotiated meaning about average and instantaneous rate laws before constructing their solution pathway with predictive and verificative arguments in terms of CMR of the situation from the problem statement. The following arguments leading to the nature of solution they provided are presented below. Here the focus is on predictive and verificative arguments and the type of justification each student (e.g. S1, S2, etc.) offers for his/her CMR or IR:

1. S3: The rate law for the first order process is  $A \rightarrow P$
2. S7: There, you probably denoted  $p$  as products, right?
3. S3: Correct...or do you see anything wrong with that?
4. S7, S44: ...why not write it in full as  $A \rightarrow \text{Products}$  ?
5. S19: Before we continue let's go back to the problem statement, what does the question say?
6. S3: ...okay, it says we should determine if the concentration of the reactant by first order kinematics
7. S6: Underline "first order kinematics" so that we don't forget, for the average rate...

$$\text{Rate (Ms}^{-1}\text{)} = -\frac{\Delta[A]}{\Delta t} = k[A]$$

8. S7: And...this  $= -\frac{d[A]}{dt} = k[A]$ , what is the difference?
9. S19: But we cannot equate that to average rate, it won't work or lead us anywhere, the first equation written on the right side of the board by Harry will

work for average calculation...I think that's for the instantaneous rate...

10. S7: But, by applying...

Lines 1 – 10 contain both predictive and verificative arguments. While S3 thinks that it makes no difference writing “products” as “p”, the suggestion made by S7 and S44 helps to clarify a possible incongruity because “p” could symbolise any variable. What S19 says along with S7 can be interpreted as being predictive and verificative argumentation. This type of argumentation reappeared several times during their work.

11. S7: By applying the infinitesimal changes in concentration, the rate law equation becomes...

$$= -\frac{d[A]}{\Delta t} = k[A], \text{ if you think about it.}$$

12. S3, S19, S44: ...correct, Ryan why didn't you include time in your statement to support the equation?

13. S7: Oops!...you mean I should say concentration and time?...

14. S19: Yeah, also change  $\Delta$  to  $d$  in the equation, so that the instantaneous

$$\text{Rate} = -\frac{d[A]}{dt} = k[A]$$

15. S7: Doing so, we integrate in terms of...

16. S44:  $d[A]$  and  $dt$ ...

17. S3: ...that will lead to ...  $\int_{[A]_0}^{[A]} \frac{d[A]}{[A]} = -k \int_0^t dt$

18. S7: So then,  $[A] = [A]_0$  or am I missing anything out?

19. S44: Yeah, for that to be so we need to state that time  $t = 0$ ...for initial reaction...only then you are in order...

20. S3: Louis, he also need to state that  $[A] = [A]$  at time  $t = t$

After reaffirming what the problem statement says (lines 6 – 7), S7 went on to reinstate his proposed equation. He provided a justification for his claim (line 11). His peers (S3, S19, and S44) challenged his assertion and warrant (line 12). This was followed up by verificative argumentation (lines 12-14): for example, they referred him to the missing variables in his proposed equation. The reasoning of these students is classified as CMR since it is novelty (i.e. a new reasoning sequence is created or a forgotten one is recreated) and is based on verificative (line 14).

21. S19: Next, we have  $\ln\left(\frac{[A]}{[A]_0}\right) = -kt$ , right?

22. S7: Can we not say this  $\int_{[A]_0}^{[A]} \frac{d[A]}{[A]} = -k \int_0^t dt$  leads to  $\ln\left(\frac{[A]}{[A]_0}\right) = -kt$ ?

23. S19: ...doesn't matter how you get there...I knew it, but didn't bother...

24. S7: I was only drawing our attention to the relevant of the former equation...

25. S3: Ryan is correct, we can then take the exponent to each side of the



equation...

26. S19: Okay...it follows as  $\frac{[A]}{[A]_0} = e^{-kt}$

27. S44: ...or  $[A] = [A]_0 e^{-kt}$

28. S6: We conclude by saying yes to the question...that the concentration of a reactant governed by the first order kinematics falls off from an initial concentration exponentially with time

29. S19, S44, S7: Yeah.

Lines 21 – 29 exemplified how the students followed their solution pathway to arrive at conclusion. The integration, in terms of  $d[A]$  and  $dt$  expressed by S3 (line 17) was revisited by S7 (line 22) when he questioned the underlying assertion of the equation. An unspoken warrant exists that connects his data to his claim: for example, he was of the opinion that the particular CMR being used by S3 should indicate what mathematical tenet it supports. Thus, he frames his reasoning in terms of imitative reasoning (IR), i.e. he corroborated his assertion from knowledge resources made available by S3. They then initiated an attempt to elaborate on the equation. This led them to an episode of verificative argument from which they concluded their problem solution.

### **Relationships Between Students' CMR and IR and Predictive and Verificative Arguments**

Students used CMR to construct predictive and verificative arguments to enable them propose solution to the problem. They related common use of rules and definitions in calculus and chemistry: in some cases they simply make connections for convenient purposes. In lines 16 – 20, they pointed to the relevant features of integration they previously drawn on the interactive whiteboard. Explicitly, they were debating whether or not more mathematical statements are needed for S7's statement to be in order (line 18). Sensing that he could be doing something wrong or overlooking some variables of the equation he proposed, S7 seeks his peers indulgence "...am I missing anything out"? (line 8). This led to various framing and reframing of S7's equation, as a result of which S44 and S3 pressed for certain conditions to be met:  $[A] = [A]_0$  at time when  $t = 0$  and  $[A] = [A]$  when  $t = t$ . This indicates a relationship between predictive and verificative arguments characterised by CMR over the useful way to frame chemistry terms in mathematics. In their solution pathway to reach a possible consensus, they showed responsibility and a sense of ownership to their own learning when they resolved a conflict between S7 and S19 (lines 23 – 25). This is one of the most important observations in this study indicating that these students were able to solve disagreements among themselves. Finally, possible decision point was reached at which the students concluded that the concentration of a reactant governed by the first order kinematics falls off from an initial concentration exponentially with time  $[A] = [A]_0 e^{-kt}$ .

Another interesting result came from a group of 6 students working out a possible solution to item #7. They had initially solved some subtasks, but did not solve the main task. Their initial efforts to find a path to a solution showed that they utilised both predictive and verificative arguments. The problem they were considering reads:

**Item #7:** A certain reaction proceeds through  $t$  first order kinetics. The half-life of the reaction is 180 s. What percent of the initial concentration remains after 900s?

30. S52: Who is doing the write up for us?

31. S30: I...from here for this,  $[A] = \frac{[A]_0}{2}$  that's the half-life.

32. S11: We should apply natural log to both sides...this should be  $\ln[A] = \ln[A]_0 - kt$ , isn't that so?

33. S61: No, no...why do we need all that? This  $\ln[A] = \ln[A]_0 - kt$  is longer path to solve the problem...

34. S28: Wait...wait...what's the problem? I just think we must apply the half-life equation directly to solve the problem.

35. S1: ...that's  $t_{1/2} = \frac{\ln 2}{k} = \frac{0.693}{k}$

Looking at the students' on-going arguments, they did not try to understand why all the equations they were considering linked to the one proposed by S1 (line 35). Instead they continued making predictions, such that each time an idea is created, they abandon it without trying to understand why it does not fit into the problem situation. After what looked like a sense of frustration, they managed to crack the code of novelty characterised with CMR and how it can be applied to solve the main task. With this, a lot of pieces in the puzzle fell into place, and predictive and verificative arguments became possible.

36. S28: With the reaction half-life equation, we need to find the rate constant  $k$

37. S1, S52: Right!

38. S30: Then,  $k = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{180}$ , put that in calculator...please

39. S1: Uhhh...there we have it,  $k = 0.00385s$

40. S30: Is  $k$  value expressed in seconds, look...  $k = 0.00385s$ ?

41. S1: Oops! My mistake,  $k = 0.00385s^{-1}$

42. S28 ...first part done! Next, we are to find what percent of  $[A]_0$  that remains after 900s.

43. S61: I think we must use the integrated rate law

44. S30, S11: Can't we just substitute the value of  $k = 0.00385s^{-1}$  and  $t = 900s$ ?

45. S1: To that we make use of the equation we derived in item #2, that says  $\frac{[A]}{[A]_0} = e^{-kt}$ .

46. S30: Yes, we are in order.

47. S30:  $\frac{[A]}{[A]_0} = e^{-kt} \rightarrow \frac{[A]}{[A]_0} = e^{-0.00385s^{-1} \times 900s}$ , help again with calculator...

48. S61: I've got ( $0.0312$ )...please somebody else should confirm the answer...

49. S52: ...correct, that is about 3.12%.

50. S28: Yeah, since the ratio of  $[A]$  to  $[A]_0$  represent the fraction of  $[A]_0$  that remains... it's 3.12%.

There is an intense framing predictive and verificative arguments going on in lines 36 – 50 as students work their solution pathway. S30 attempts to reframe the discussion (line 40). She points to the value of  $k$  they've written incorrectly and asks, "Is  $k$  value expressed in seconds, look...  $k = 0.00385s$ ?" She calls the attention of her peers to evaluate the unit expression of  $k$ . An unspoken warrant in this case is that she must have compared the unit expression of time  $t$  measured in seconds with that of  $k$  measured in per second ( $s^{-1}$ ). S1 responds in a familiar way to S30's bid for correction. Correction was made, resulting in an efficient conversation on what type of CMR justification should count. It was a prelude to predictive argument when S1 certified the valid use of this equation  $\frac{[A]}{[A]_0} = e^{-kt}$  (taken from item #2) to a more familiar item #7 they are considering. He was, after all, the one who actually wrote the half-life equation that helped to guide his peers in the right direction of solution pathway (line 35). Upon affirmation, their problem-solving operation focuses on how valid uses of chemical kinematics variables align with calculus framing in terms of CMR. The verificative argument relies on another kind of warrant (lines 48 and 49). From this, S28 made a plausible explanation, a hybrid of the calculation to which he invokes a warrant that he sees as most appropriate (line 50).

### **Relationships Between Students' CMR and IR and Predictive and Verificative Arguments**

The use of CMR by the students in solving item #7 was rarely consistent with predictive and verificative arguments pertaining to the activity. Although they use strategy for recalling memorized facts and procedures and occasionally framed calculus terms correctly, their engagement in terms of sense making imply less of a necessity for argumentation, which is characteristic of imitative reasoning (IR). It seems that the lack of argumentation disqualifies the reasoning as creative mathematics reasoning (CMR), resulting in which the lack of predictive and verificative arguments classify the reasoning as IR. This in turn is one reason for their earlier choice to implement  $\ln[A] = \ln[A]_0 - kt$  as a feasible equation to resolve item #7 (line 32). But as long as they stay true to the chemical kinematics calculus-based rules, they were able to steer their way to a reasonable solution.

### **Conclusion and Suggestions**

Results from this study show that differences in students' reasoning and utilization of argumentation skills for use in solving chemical kinematics calculus-based problems can be referred to different characteristics of reasoning. For example, the differences among students' reasoning frequently included predictive and verificative arguments, while some did not include CMR. Upon a further look into the characteristics of the solutions that students provided, it becomes apparent that those who elaborate on warrant for counterarguments and present verificative argumentation which support claims in solutions are those who explicitly provided justifiable solutions to the chemical kinematics calculus-based problems.

What the successful students in this study all have in common is that their reasoning is

essentially characterized as being creative mathematics reasoning (CMR), path of their reasoning includes valuable framing evidence and justifications for their claims. On the other hand, there were students who failed to solve one-third of the tasks correctly. In this sense, they were able to solve sub-problems but made less use of their experiences from solving the main tasks. As can be seen in this study, utilizing argumentation skills for solving chemical kinematics calculus-based problems means framing predictive and verificative arguments that support the solution. Since management of procedures is taken care of by CMR and IR, algorithmic reasoning is essential accessory for framing and assimilating calculus rules (including making conclusions supported by verificative argumentations). Based on the findings of this study, if a purpose is to engage students in fostering reasoning and argumentation skills for use in problem solving, the antidote to students who meet difficulties includes stepwise instructions: instead of explaining how to solve a particular problem, the educator/instructor should ask the students to justify their solution methods and solutions. Finally, whilst a course in calculus may not be a formal prerequisite in many undergraduate chemistry courses, students taking general chemistry would benefit from a conceptual understanding and basic facility with derivatives and integrals and their applications.

### References

- Azarang, Y. (2012). Quality of Learning Calculus in Iran. *Roshd Mathematics Education Journal*, 27(1): 24- 30.
- Bain, K., & Towns, M. H. A. (2016). Review of Research on the Teaching and Learning of Chemical Kinetics. *Chemistry Education Research and Practice*, 17(2), 246–262, DOI: 10.1039/C5RP00176E.
- Berland, L., & McNeill, K. (2010). A learning progression for scientific argumentation: Understanding students' work and designing supportive instructional contexts. *Science Education*, 94, 765-793.
- Carius, A. C., Júnior, R. L. S., & Silva, J. M. (2017). Differential Calculus and Chemistry: the Challenge of an Interdisciplinary Course.
- Diwu, C., & Ogunniyi, M.B. (2012). Dialogical argumentation instruction as a catalytic agent for integrating science with Indigenous knowledge systems. *African Journal of Research in mathematics, Science and Technology Education*, 16(3), 333-347.
- Festus, C., & Ekpete, O. A. (2012). Improving Students' Performance and Attitude towards Chemistry through Problem-Based-Solving Techniques (PBST). *International Journal Academic Research in Progressive Education and Development*, 1(1), 167-174.
- Fraenkel, F.J. & Wallen, N.E. (2009). How to Design and Evaluate Research in Education (7th ed.). Qualitative Research McGraw-Hill Higher Education.
- Haciomeroglu, E. S., Aspinwall, L., & Presmeg, N. C. (2010). Contrasting cases of calculus students' understanding of derivative graphs. *Mathematical Thinking and Learning*, 12(2), 152–176.
- Hashemi, N., Abu, M. S., Kashefi, H., & Rahimi, K. (2014), Undergraduate students' difficulties in conceptual understanding of derivation, *Procedia – Social and Behavioral Sciences*, 143, 358-366.

- Jonassen, D.H. (2007). What makes scientific problems difficult? In D.H. Jonassen (Ed.), *Learning to solve complex, scientific problems*, 3-23. Mahwah, NJ: Lawrence Erlbaum Associates.
- Kapon, S., & diSessa, A. (2012). "Reasoning Through Instructional Analogies," *Cognition and Instruction* 30(3), 261-310.
- Kurt, S. & Ayas, A. (2012). Improving Students' Understanding and Explaining Real Life Problems on Concepts of Reaction Rate by Using a Four Step Constructivist Approach. *Energy Education Science Technology Part B: Social Education Studies*, 4, 979– 992.
- Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics*, 67(3), 255 – 276.
- Means, L.M., & Voss, J.F. (1996). Who reasons well? Two studies of informal reasoning among children of different grade, ability, and knowledge levels. *Cognition and Instruction*, 14(2), 139-178.
- Patel, V. (2012). *Chemical Kinetics* (InTech, Rijeka, 2012).
- Perkins, D. (1992). *Smart schools -from training memories to training minds*. New York: The Free Press.
- Toulmin, S. (2003). *The uses of argument*, Cambridge, England: Cambridge University Press.
- Voss, J. F. (2006). Toulmin's model and the solving of ill-structured problems. In D. Hitchcock & B. Verheij (Eds.) *Arguing on the Toulmin Model: New Essays in Argument Analysis and Evaluation*, 303–311. Berlin: Springer.
- Walker, J. P., & Sampson, V. (2013). Learning to argue and arguing to learn: Argument-driven inquiry as a way to help undergraduate chemistry students learn how to construct arguments and engage in argumentation during a laboratory course. *Journal of Research in Science Teaching*, 50(5), 561–596.
- Zhdanova, A. O., Kuznetsov, G. V., Legros, J. C., Strizhak, P. A. (2017). Thermal Conditions For Stopping Pyrolysis Of Forest Combustible Material And Applications To Firefighting. *Thermal Science*, 21(6A), 2565 – 25